Integrated and collaborative routing problems

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The framework

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Directions in routing problems

Collaborative

Integrated

Data-driven

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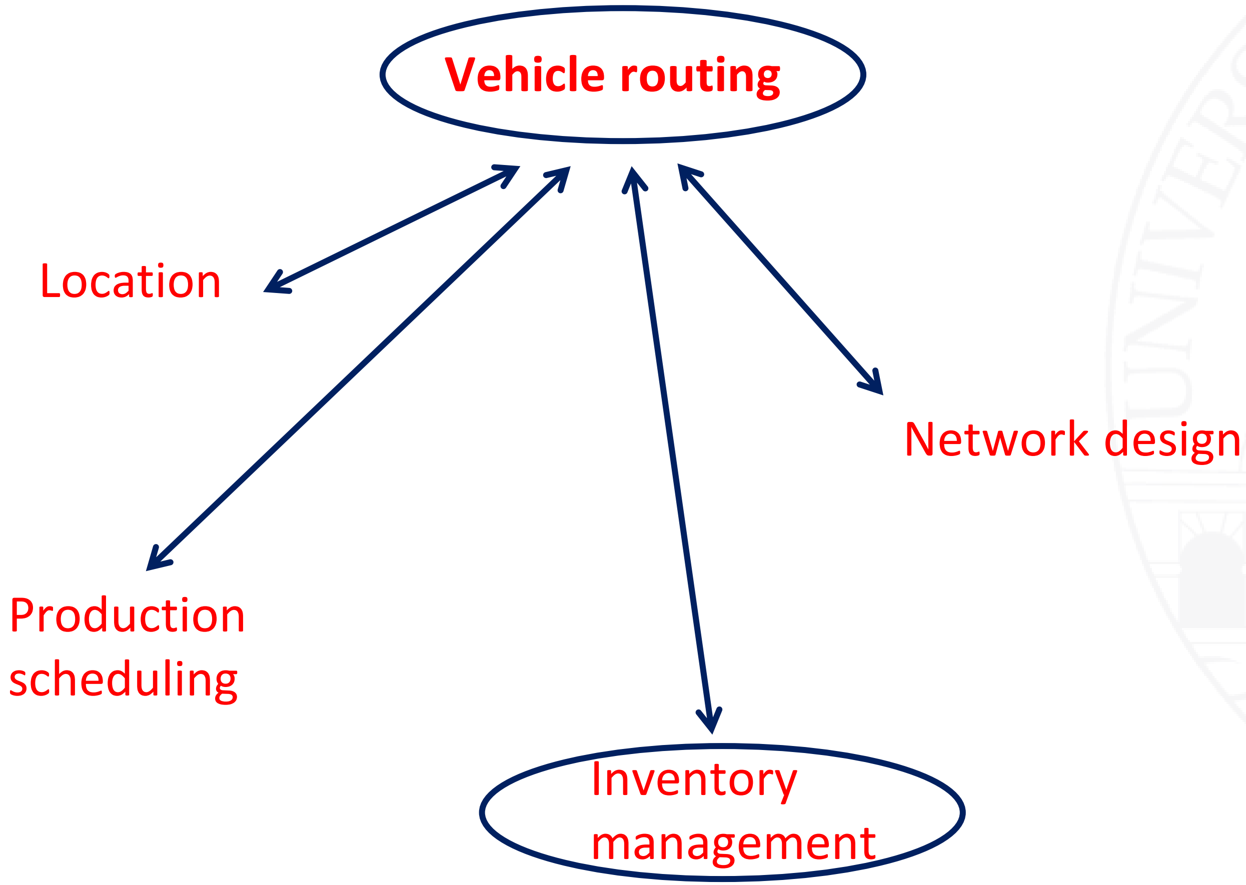
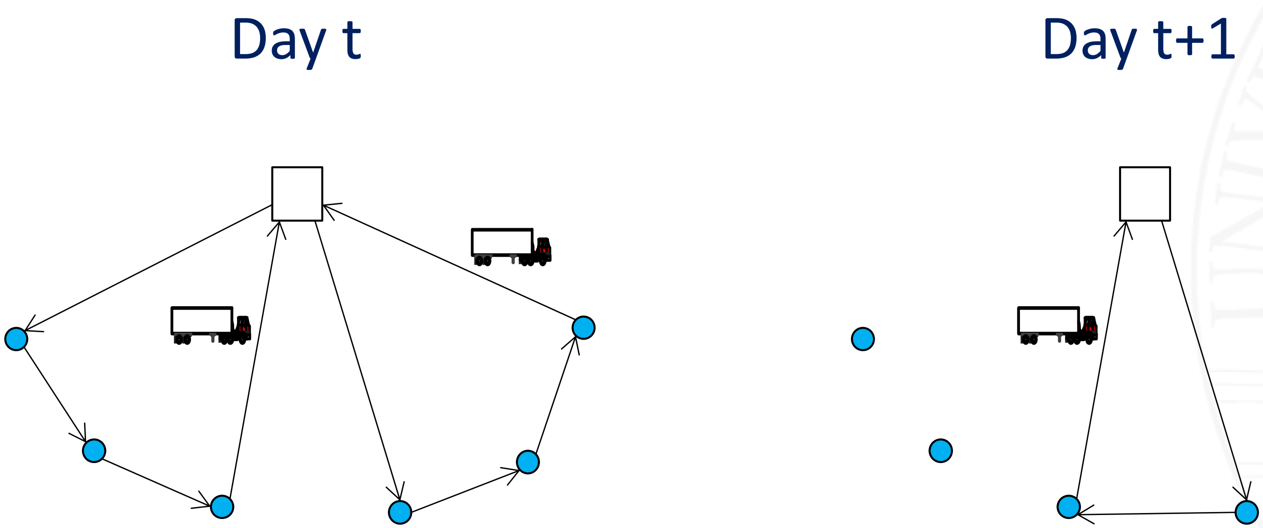
Integrated direction



Inventory routing problems

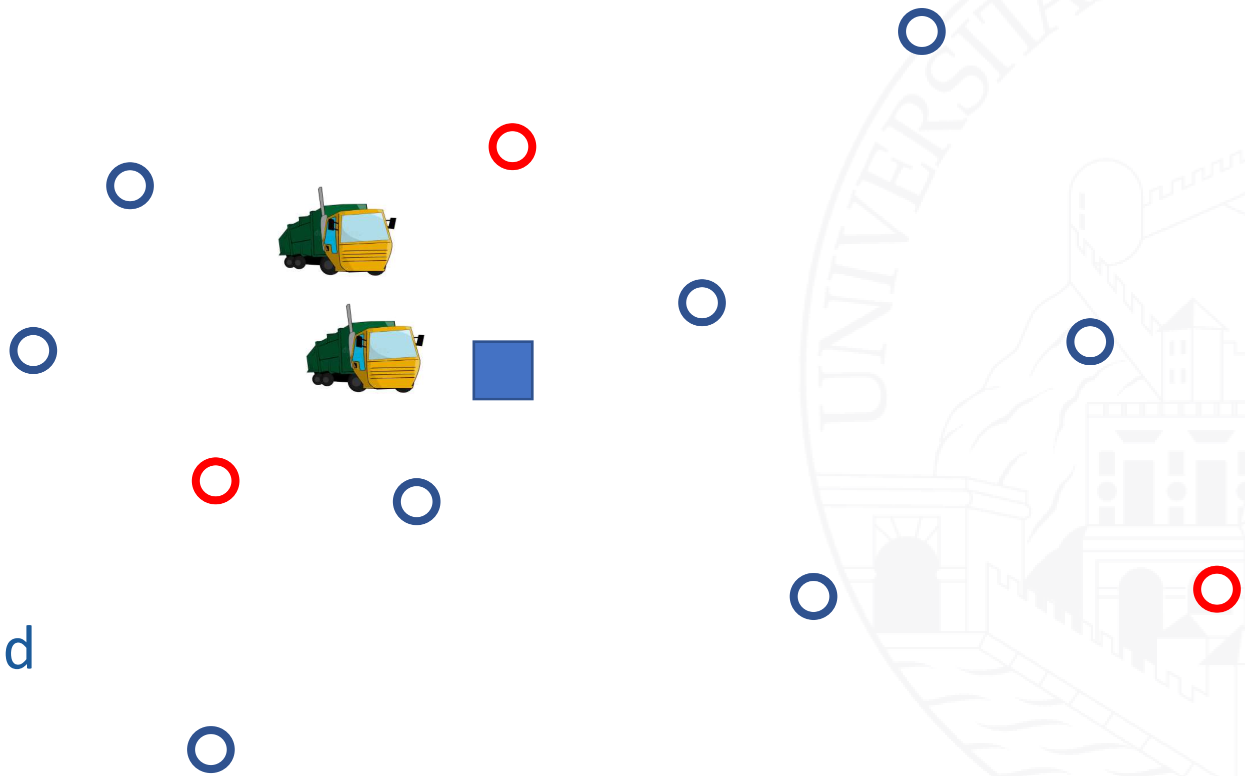


Routing problems over time

Pick-up and delivery inventory routing problem

Pick-up

Delivery

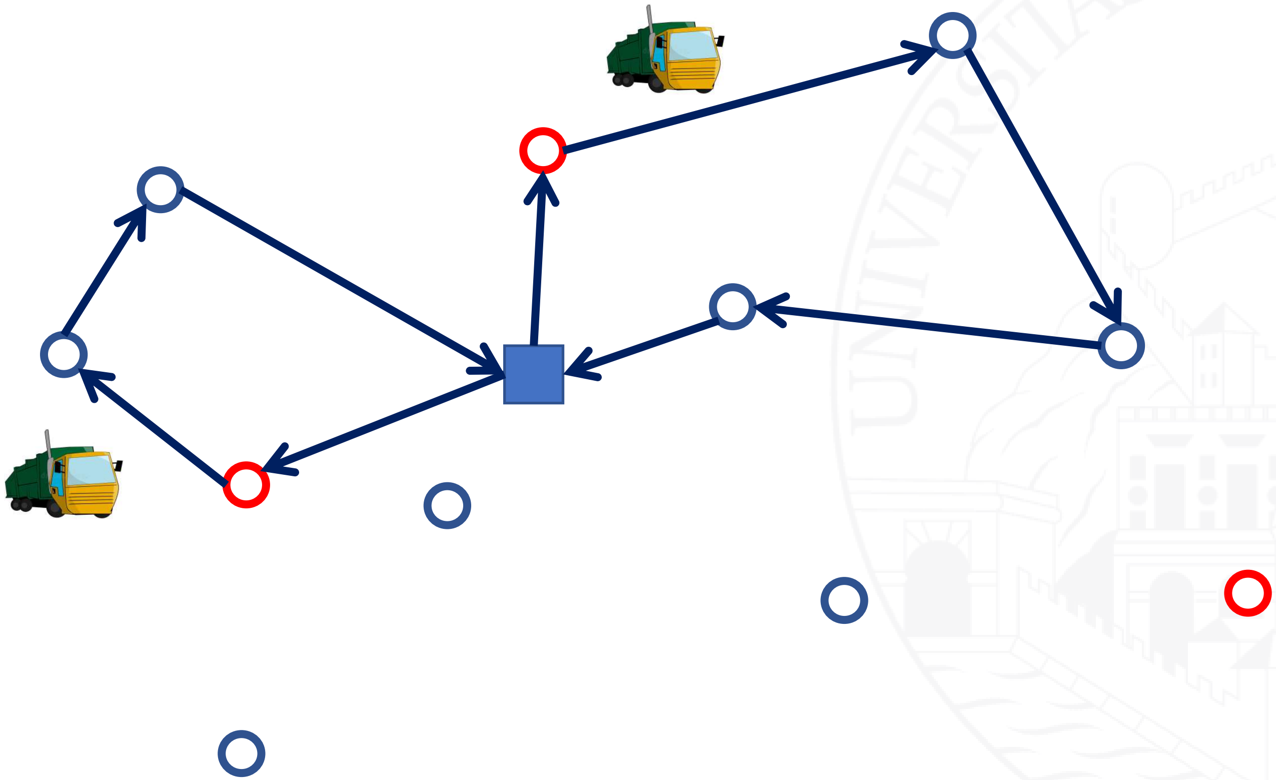


Given
availability and demand
over time

Archetti, Christiansen, Speranza, EJOR, 2018

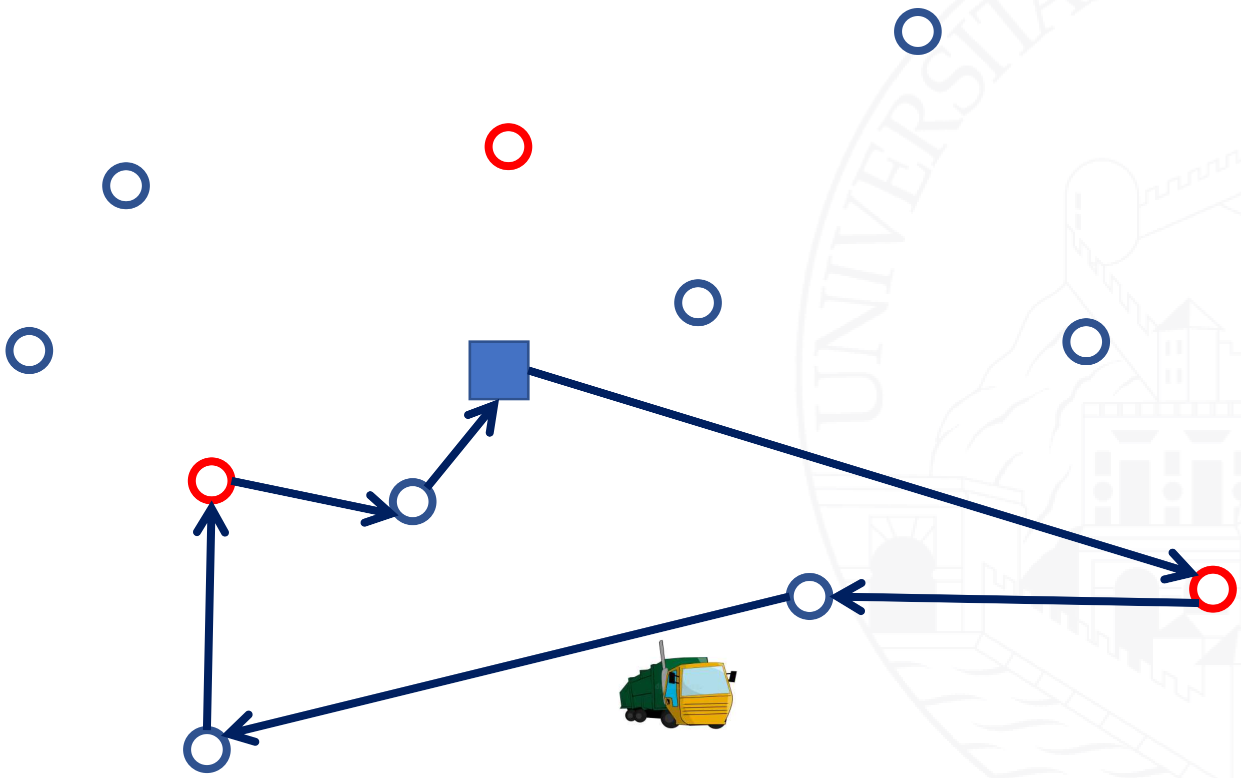
Pick-up and delivery inventory routing problem

Day t



Pick-up and delivery inventory routing problem

Day t+1



Pick-up and delivery inventory routing problem

- Pick-up customers – daily quantity made available
- Delivery customers – daily demand
- One vehicle with capacity Q
- Maximum and minimum inventory level at customers
- The depot is a warehouse where goods can be stored

Min routing cost + inventory holding cost

Pick-up and delivery inventory routing problem

Variables:

- Quantity (horizon x customers) – continuous
- Inventory level (horizon x customers) - continuous
- Visit schedule (horizon x customers) - binary
- Edge traversal (horizon x customer^2) – binary
- Load (horizon x customer^2) – continuous

Objective function:

Min routing + inventory holding costs

Constraints:

Inventory constraints

Vehicle capacity constraints

Routing constraints

Load constraints

Pick-up and delivery inventory routing problem

640 instances with varying:

- vehicle capacity: $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$
- horizon: 3 or 6
- inventory cost: high or low
- number of customers: up to 50

473 instances solved to optimality

1.22 average optimality gap

Improved branch-and-cut algorithm

Archetti, Boccia, Sforza, Speranza, Sterle, submitted

538 instances solved to optimality

0.89 average optimality gap

133 improved solutions

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Pick-up and delivery inventory routing problem

- Integrated policy
- Sequential policy: each delivery customer applies (s,S)
  vehicle routing problems

	% total cost (average)	% total cost (max)
T=3	40.47	63.19
T=6	27.66	40.28
Low inventory cost	36.36	52.97
High inventory cost	34.67	63.19
All	35.54	63.19

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Directions in routing problems

Collaborative

Integrated

Data-driven

Routing among loading/unloading areas

Mor, Speranza, Viegas, in preparation, 2019

Routing among loading/unloading areas

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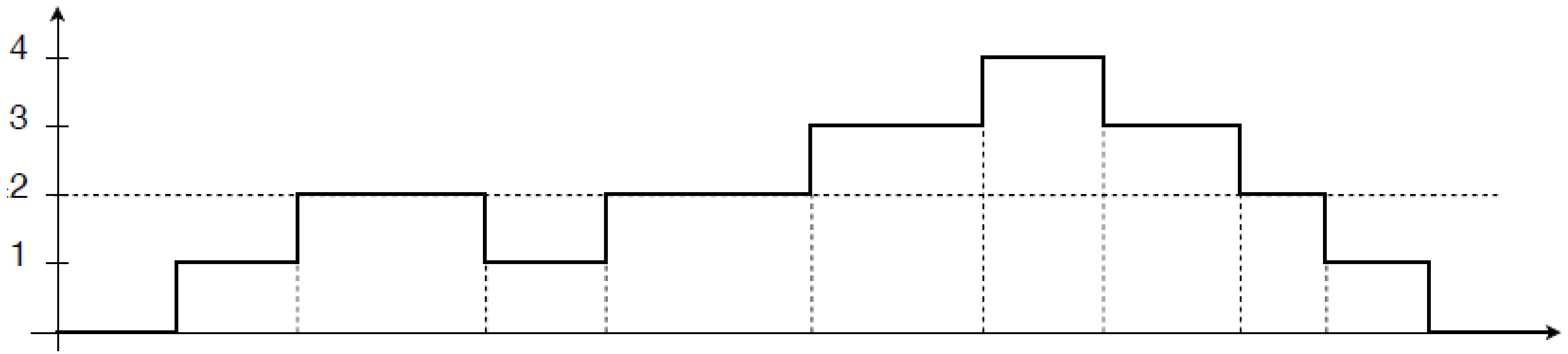
Routing among loading/unloading areas



Routing among loading/unloading areas

Each vehicle makes a reservation of the L/U areas

↓

Windows of availability for the following vehicles

↓

Variant of TSP with multiple time windows

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TSP with multiple time windows

$$\min T_{|U|+1} - T_0$$

$$\sum_{u \in U} x_{0u} = 1$$

$$\sum_{u \in U} x_{u(|U|+1)} = 1$$

$$\sum_{i \in \{0\} \cup U} x_{iu} = \sum_{i \in U \cup \{|U|+1\}} x_{ui} = 1 \qquad u \in U,$$

$$(T_i + s_i + t_{ij} - T_j) \leq M(1 - x_{ij}) \qquad i \in \{0\} \cup U, j \in U \cup \{|U|+1\},$$

$$T_u \geq W^a_{u,h} y_{u,h} \qquad u \in U, h \in H_u,$$

$$T_u + s_u \leq W^b_{u,h} + M(1 - y_{u,h}) \qquad u \in U, h \in H_u,$$

$$\sum_{h \in H_u} y_{u,h} = 1 \qquad u \in U,$$

$$x_{ij} \in \{0,1\} \qquad i \in \{0\} \cup U, j \in U \cup \{|U|+1\},$$

$$T_i \geq 0 \qquad i \in \{v_k, v_k + 1\} \cup U,$$

$$y_{u,h} \in \{0,1\} \qquad u \in U, h \in H_u.$$

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Routing among loading/unloading areas

Fixed starting time of each route

$$t_{0i} = 0 \qquad i \in U \cup \{|U|+1\}$$

$$t_{i(|U|+1)} = 0 \qquad i \in \{0\} \cup U$$

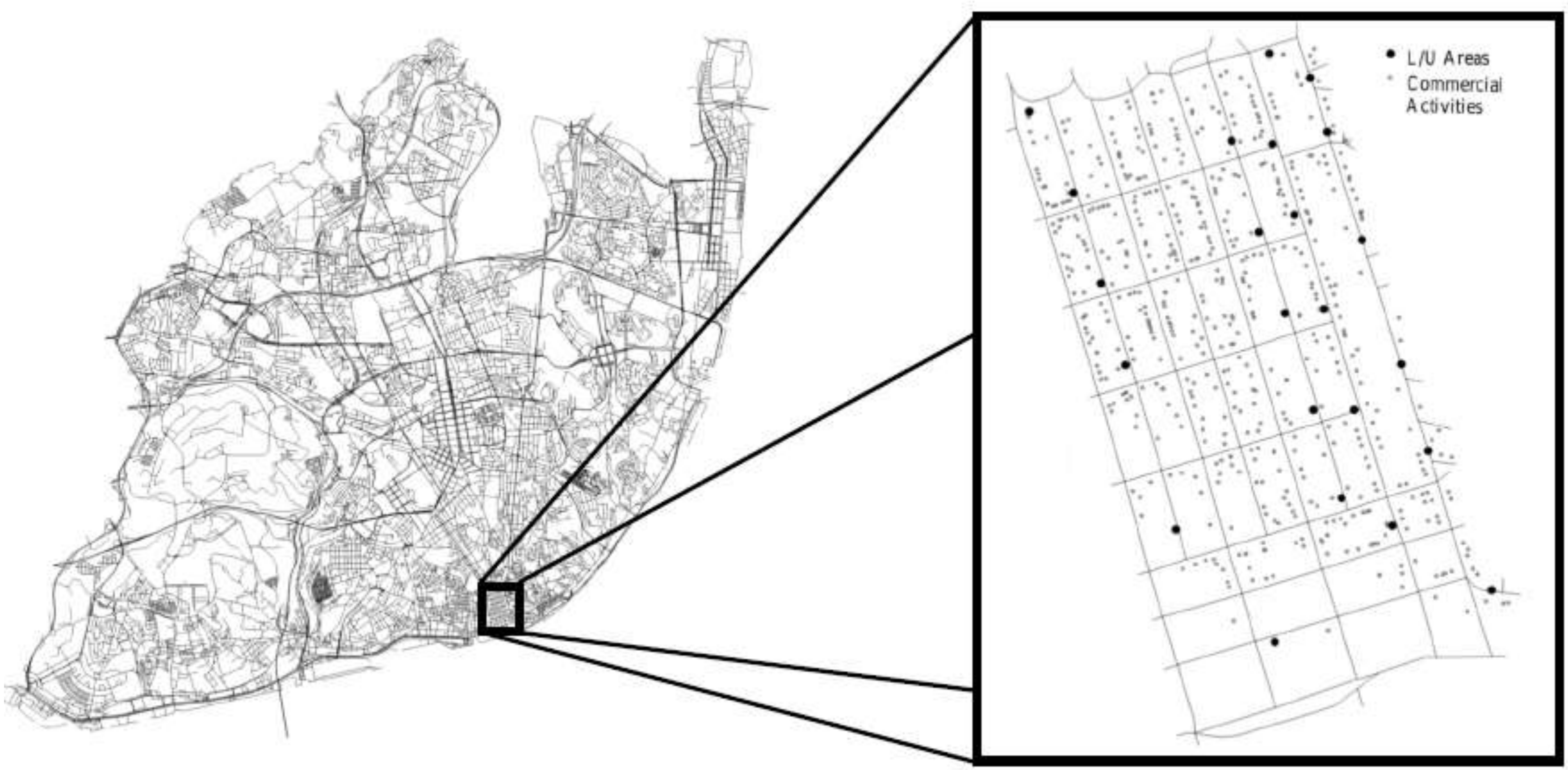
$$T_0 = 0$$

$$s_0 = h$$

Variable starting time of each route

$$s_0 = 0$$

Routing among loading/unloading areas



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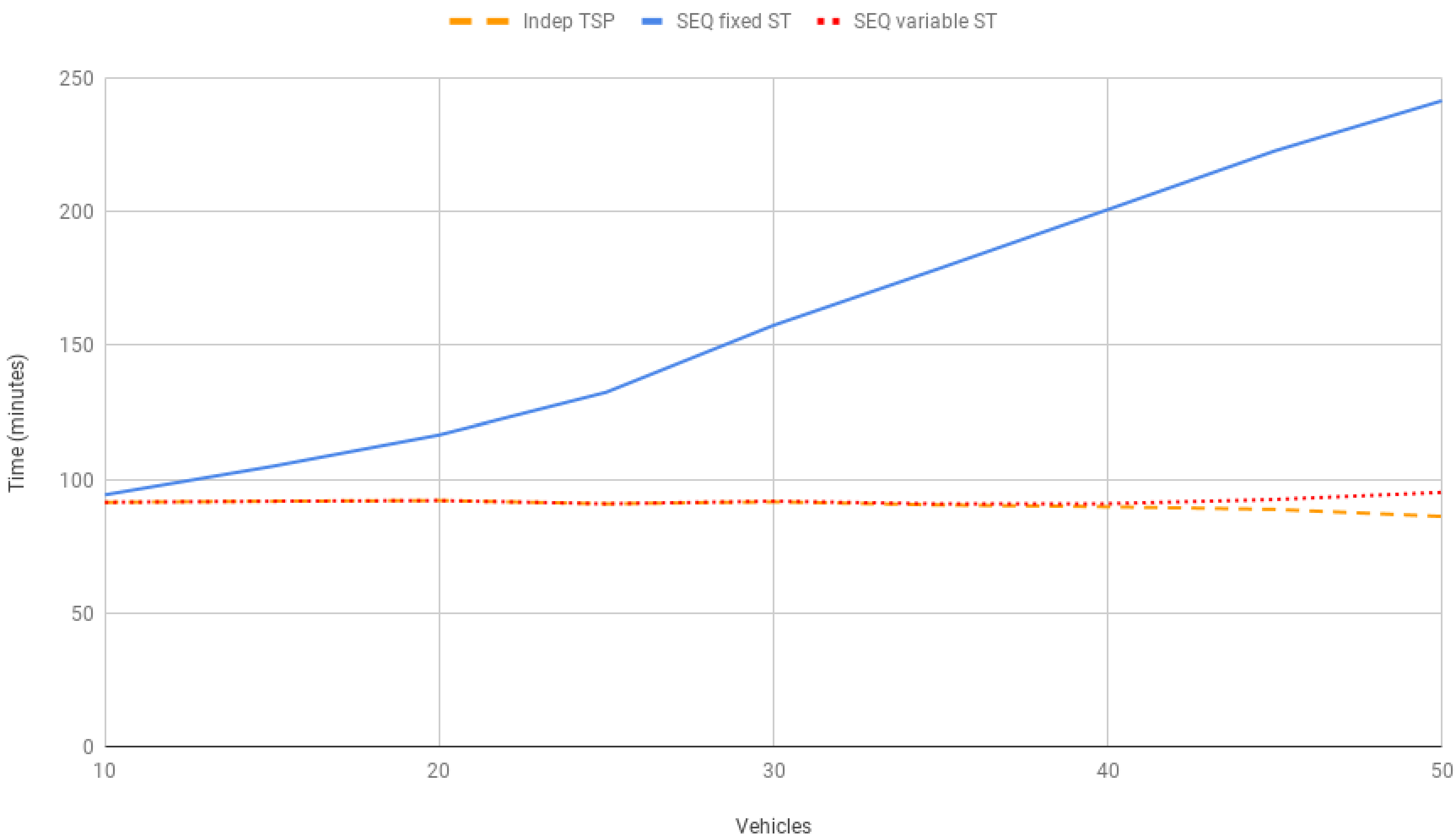
Routing among loading/unloading areas



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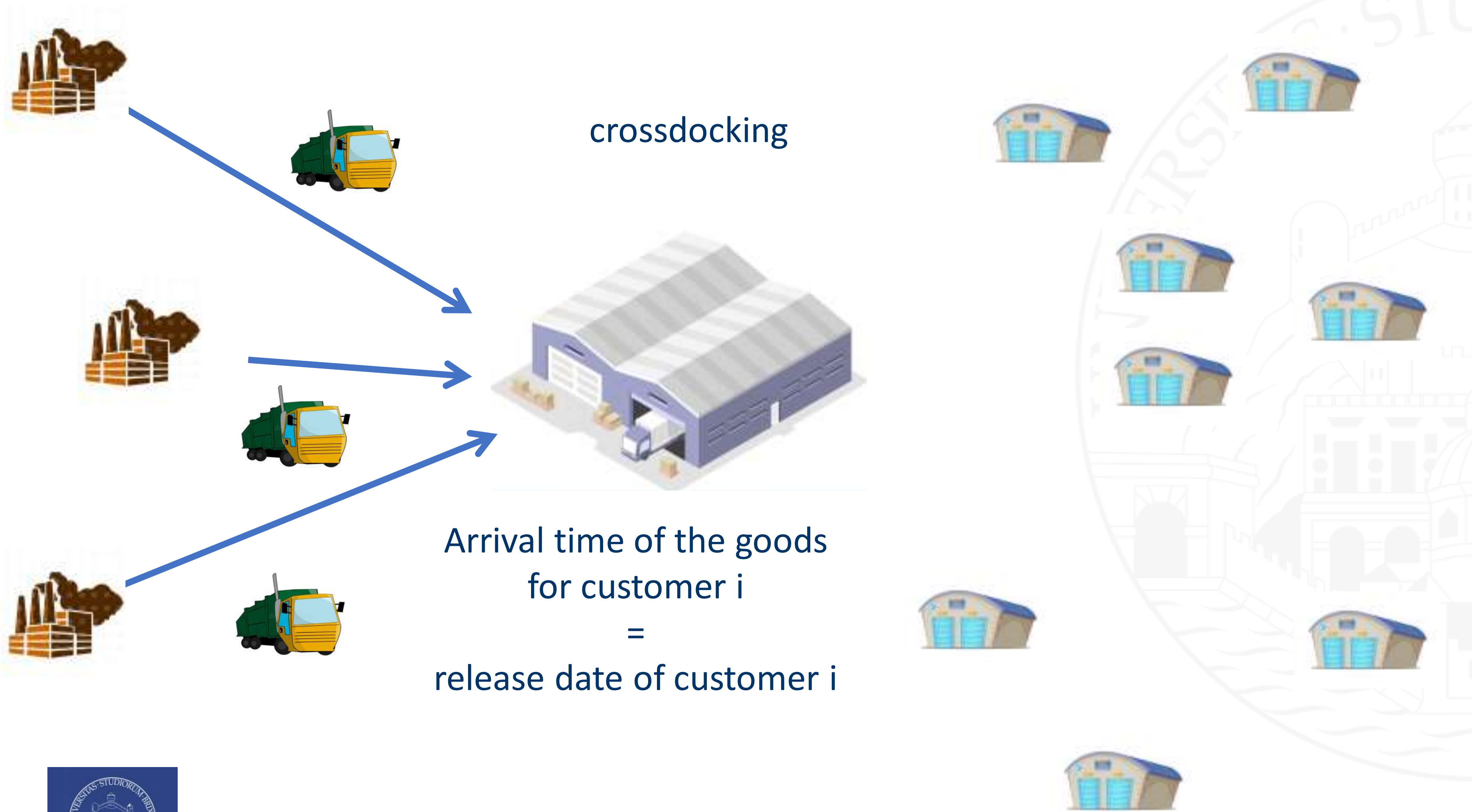
Directions in supply chain management

Collaborative

Integrated

Data-driven

VRP with release dates



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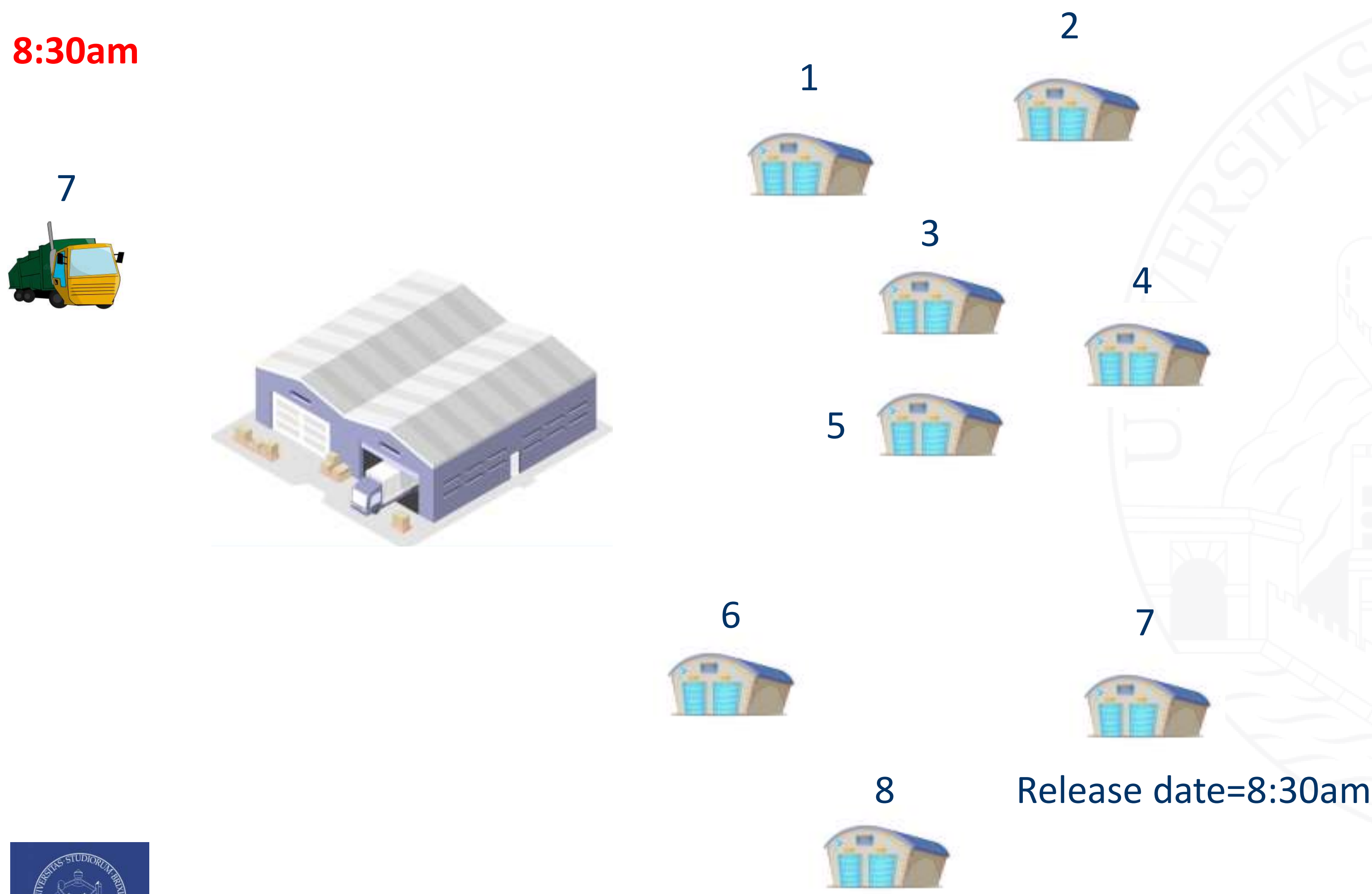
VRP with release dates



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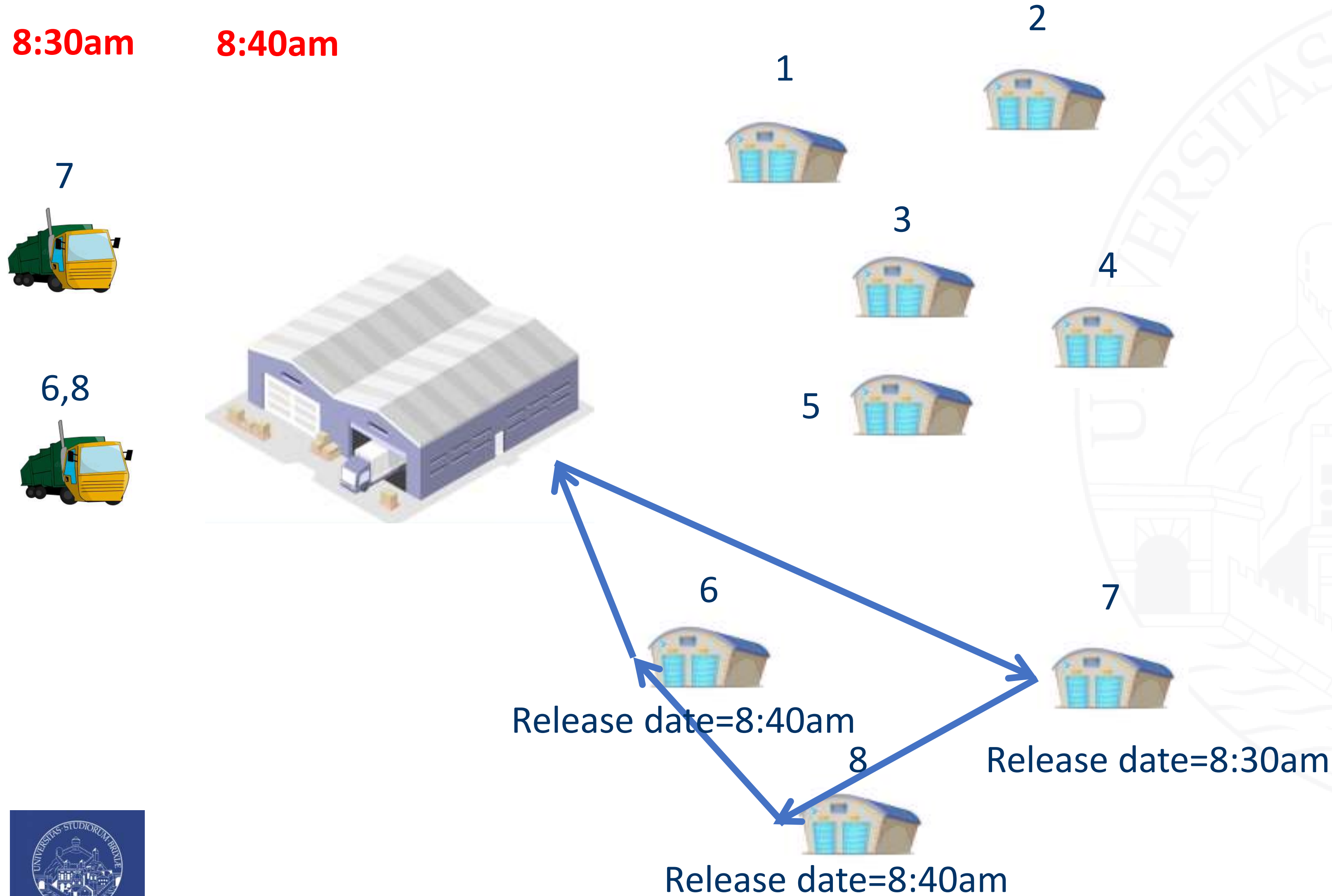
VRP with release dates



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TSP with release dates

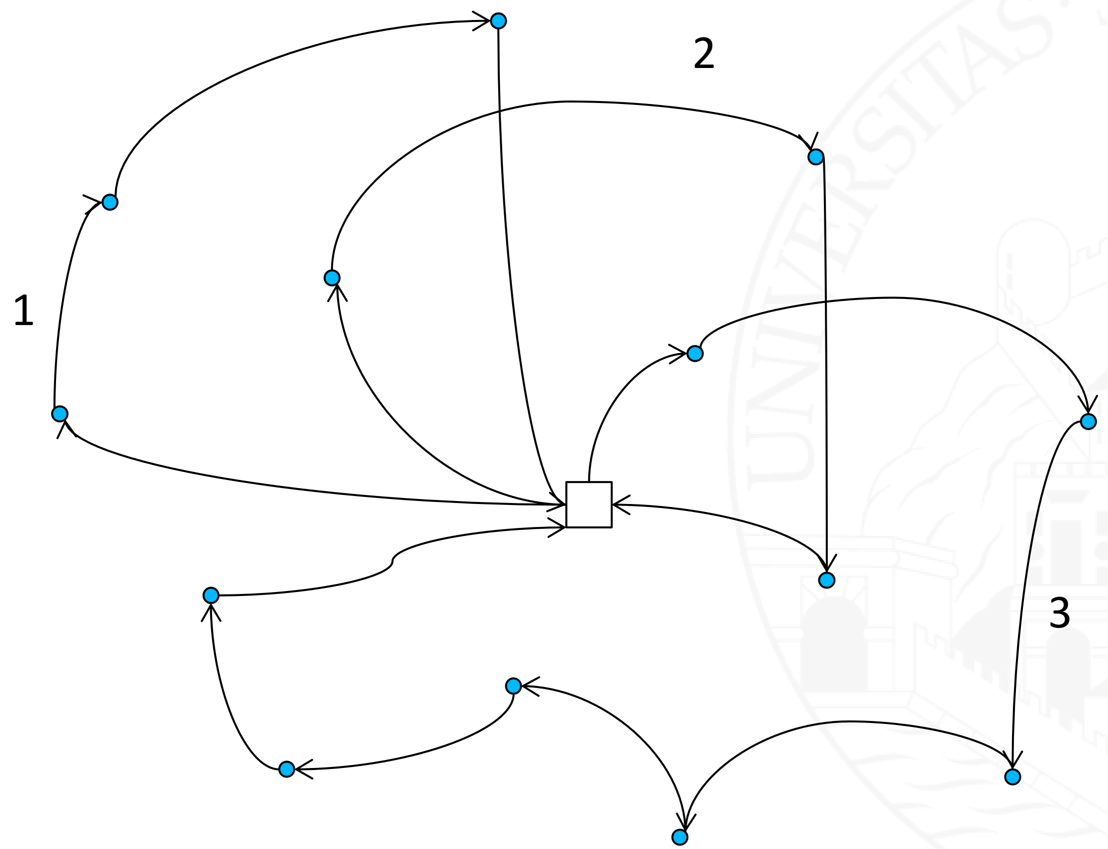
Min

Maximum completion time

=

Traveling time +

Waiting time at the depot



Archetti , Feillet, Mor, Speranza, EJOR, 2018

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TSP with release dates

Property: There exists an optimal solution without waiting time after the start of the first route

largest release date

A lower bound: $t(S^*) \geq \frac{r_n + d_{TSP}}{2}$

An approximation algorithm:

Apply Christofides' algorithm

Start the TSP tour obtained at time r_n

Performance guarantee: 2.5

TSP with release dates

n	# optimal solutions
10	24
15	24
20	24
25	13
30	7

Iterated local search

Iterated local search with a MILP operator

TSP with release dates

Myopic: visit customers when they become available

ILS	ILS-MILP	Myopic
0.01	0.86	16.12

50 customers: gaps with respect to best known solution

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TSP with stochastic release dates

Arrival time at the warehouse often not known

Information may become available over time

TSP with stochastic release dates

The release date of a customer (arrival time of a truck) may be:

- known (reliable)
- static (random but the distribution does not change over time)
- dynamic (random and the distribution changes over time)

Archetti , Feillet, Mor, Speranza, submitted

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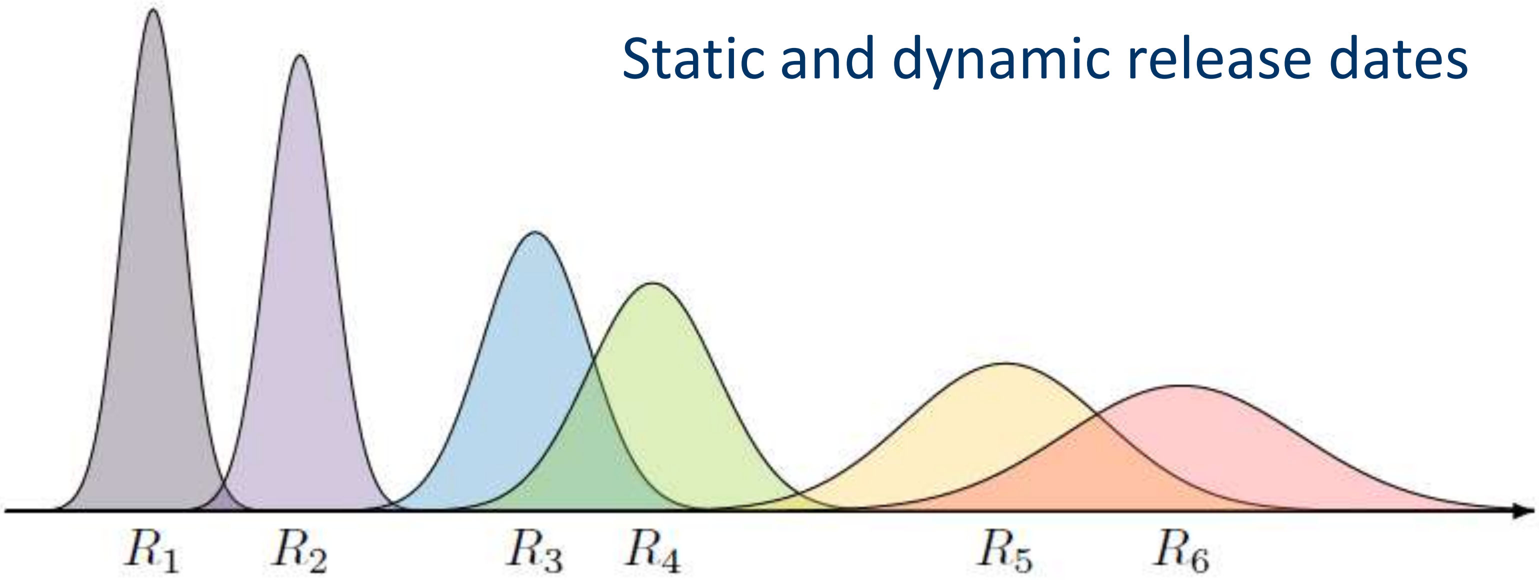
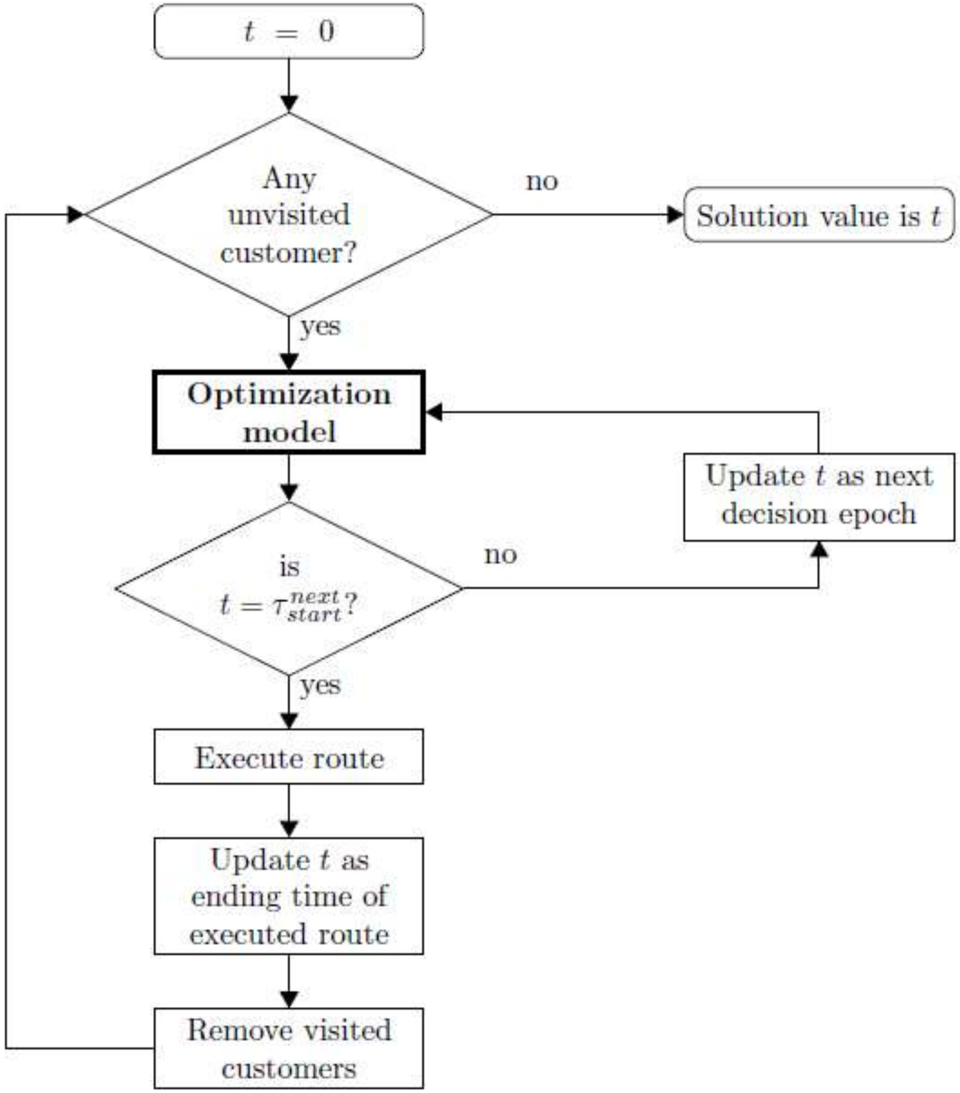
TSP with stochastic release dates

Static and dynamic release dates



TSP with stochastic release dates



TSP with stochastic release dates

Size	Deterministic			Stochastic		
	t_{LS}	n_{LS}	$\bar{t}_{LS}$	t_{LS}	n_{LS}	$\bar{t}_{LS}$
50	0.11	8	0.014	49.14	16	3.07
60	0.14	8	0.017	46.34	14	3.31
70	0.28	22	0.013	335.93	36	9.33
80	0.41	58	0.007	406.88	15	27.13
90	0.25	42	0.006	557.44	15	37.16
100	0.34	31	0.011	310.07	16	19.38

Time to perform 1 iteration of the Iterated Local Search

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Directions in routing problems

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The shared customer collaboration VRP

- Each company has its depot and fleet
- There is a subset of **shared** customers
- Companies are willing to **share** the service of some customers with other companies in order to decrease their cost

Horizontal
Between competitors
Order sharing

Fernàndez, Roca-Riu, Speranza, EJOR, 2018

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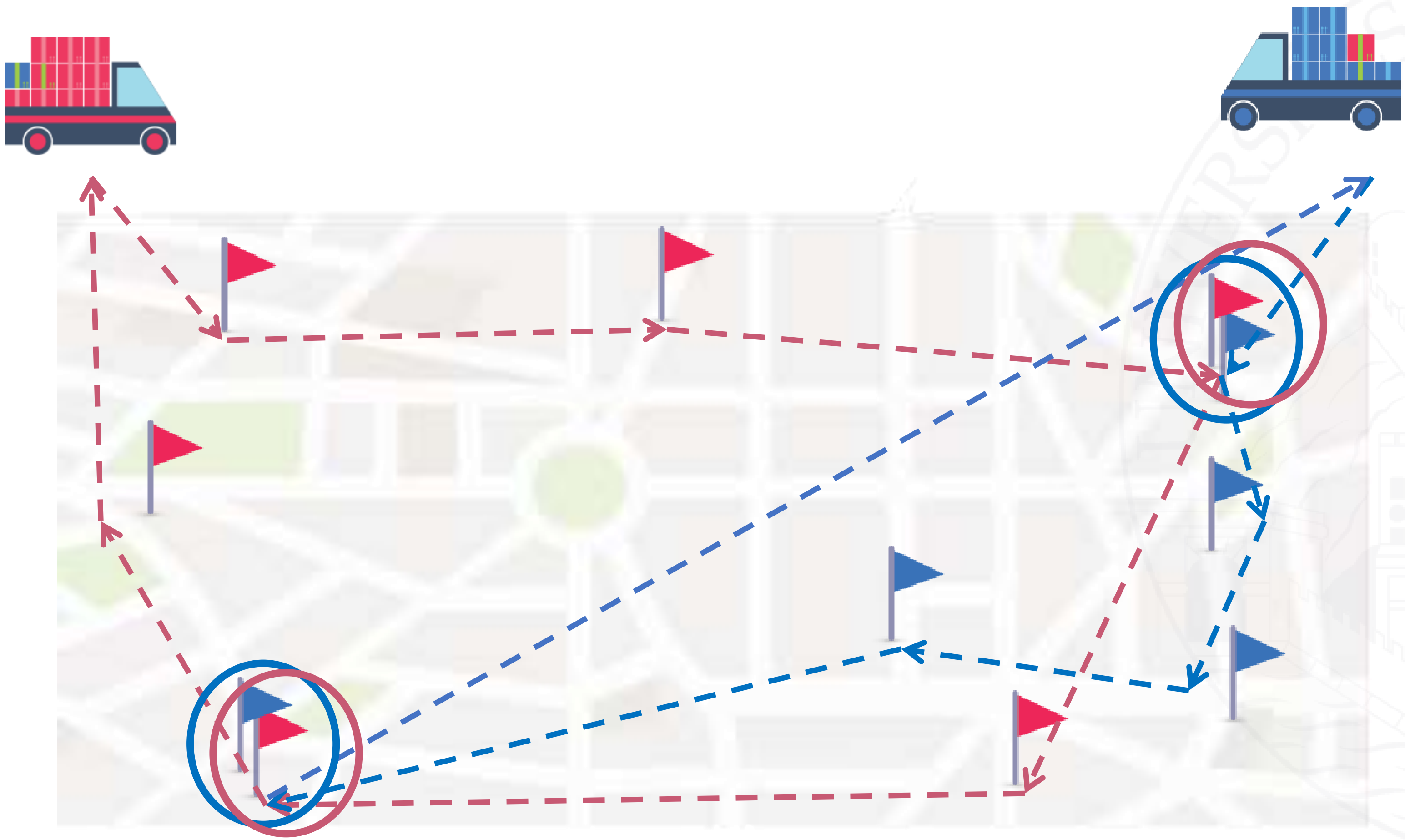
The shared customer collaboration VRP



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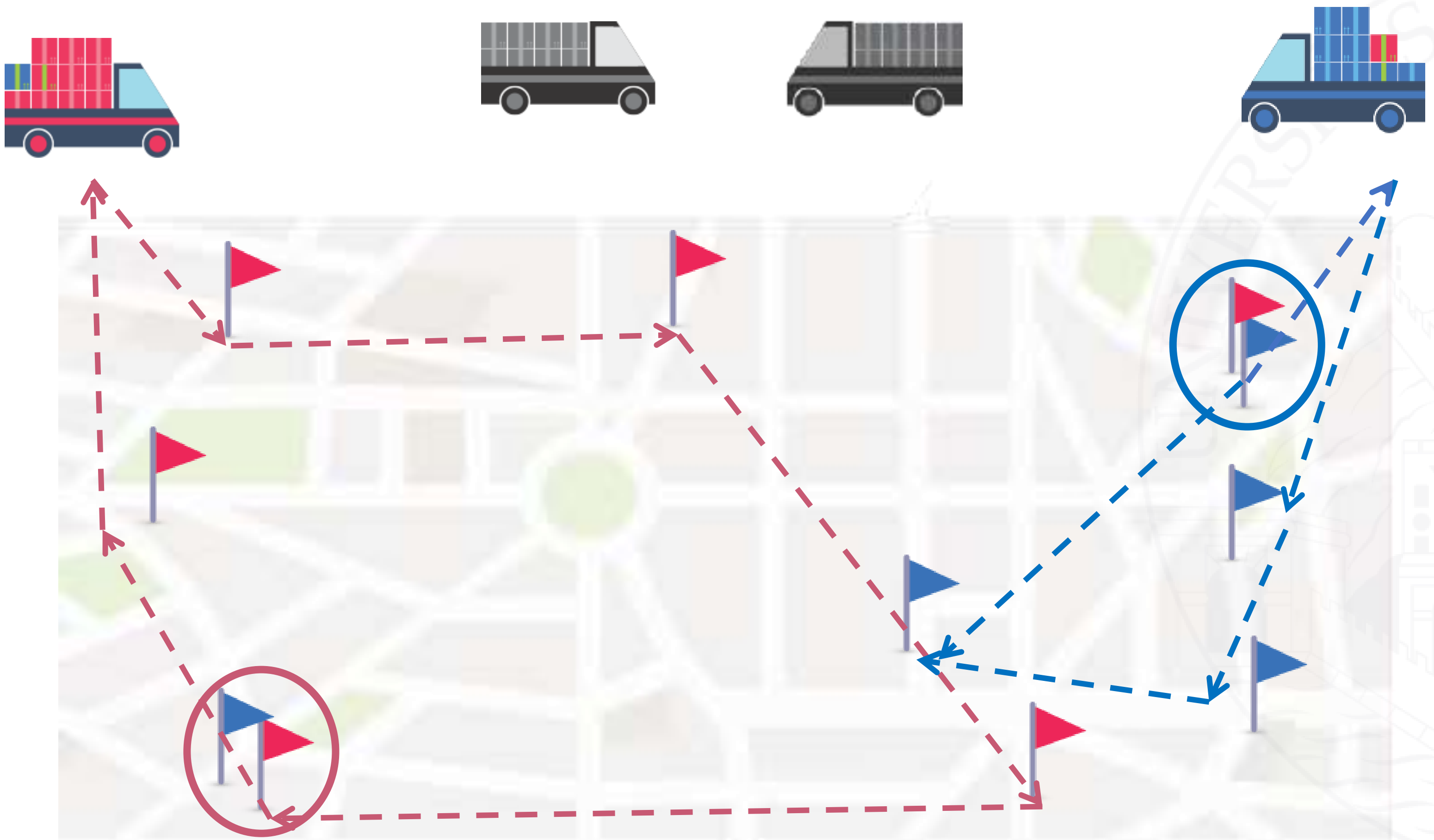
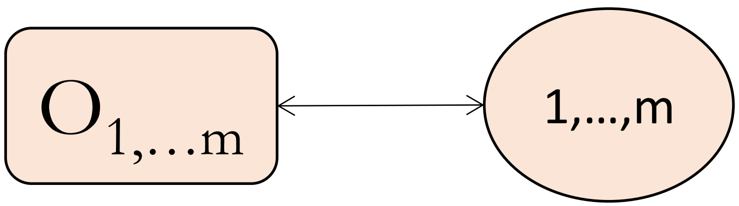
The shared customer collaboration VRP



Analysis

$$z^*(\text{SCC-VRP}) \geq \max_{r \in C} z^*(\text{VRP}_r)$$

$$z^*(SCC\text{-}VRP) \geq \frac{z^*(m\text{-}VRP)}{m}$$



All depots are co-located
All carriers have 1 co-located customer

If it is guaranteed that in the collaboration the profit of each company does not decrease with respect to any sub-coalition, the solution belongs to the core of the game

Formulations

Vehicle formulation

x_{ij}^{k} arc i,j for vehicle k

z_{irs}^{k} customer i
from carrier r to s, with vehicle k

Load formulation

x_{ij}^{r} arc i,j by carrier r

z_{irs} customer i from carrier r to s

l_{ij}^{rh} load on arc i,j by carrier r
for customer h

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Solution approach

Vehicle formulation

Branch & Cut

+ Cover inequalities
+ Capacity-cut inequalities
+ Symmetry breaking constraints

Load formulation

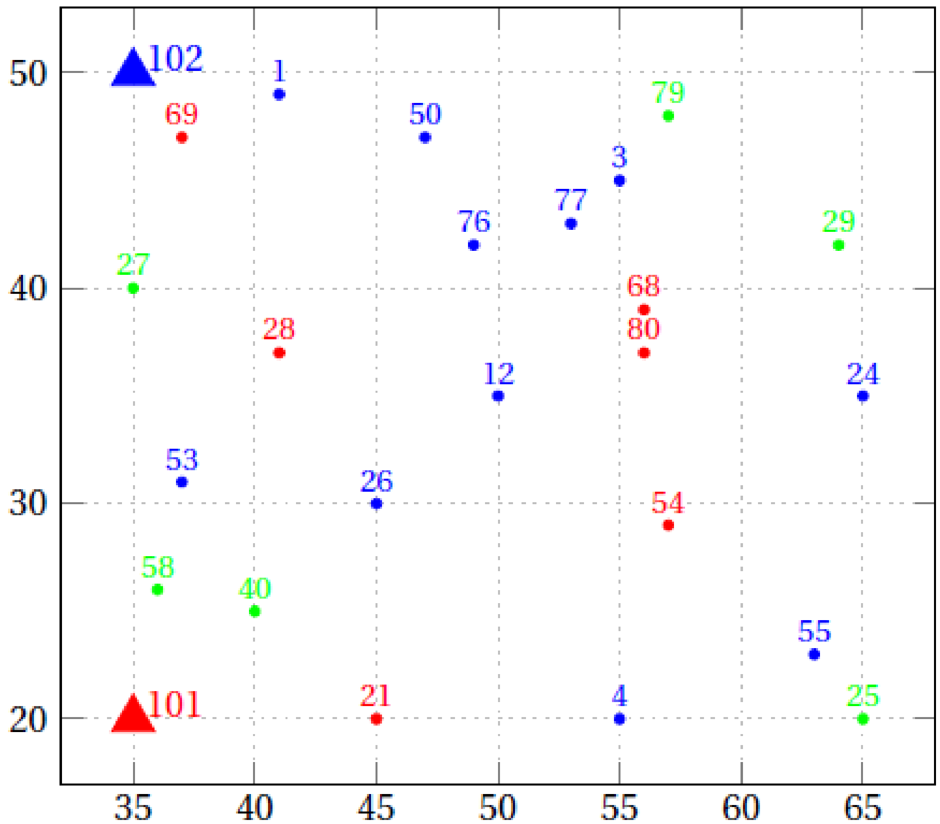
Branch & Cut

+ Connectivity constraints
+ Capacity-cut inequalities
+ Symmetry breaking constraints

Test instances

S1 – adapted from
Cordeau, Gendreau, Laporte, 1997
12 instances for the MDVRP
18-30 customers

S2 – randomly generated
100 random/clustered instances
10-30 customers



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Comparison between VF and LF (S1)

	VF					LF				
	Obj	r_A	r_B	*%Gap*	*T(s)*	*Obj*	r_A	r_B	*%Gap*	*T(s)*
1	337.45	2	2	53.44	*TL*	273.88	2	2	5.46	*TL*
2	518.13	2	2	66.05	*TL*	324.19	2	2	7.28	*TL*
3	316.78	2	2	55.11	*TL*	233.28	2	1	0.98	*TL*
4	563.58	2	2	68.00	*TL*	322.3	2	2	3.7	*TL*
5	468.54	2	2	60.60	*TL*	328.02	2	2	7.01	*TL*
6	259.87	1	2	32.68	*TL*	230.08	1	2	0	134.11
7	180.56	1	1	35.81	*TL*	156.93	1	1	0	120.1
8	536.03	2	2	65.82	*TL*	237.83	1	1	0	2472.56
9	515.48	2	2	54.12	*TL*	392.06	1	1	0	59.9
10	685.95	2	1	65.05	*TL*	455.71	1	1	0	90.19
11	494.6	1	2	20.53	*TL*	486.9	1	2	0	726.98
12	882.65	1	2	56.54	*TL*	750.6	1	2	13.00	*TL*

2 hours

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Savings

	$S2_R$ Random				$S2_C$ Clustered			
N	$\#Opt$	-%	-% A	-% B	$\#Opt$	-%	$-\%_A$	$-\%_B$
10	10	13.4	7.9	13.7	10	2.5	2.5	2.5
15	10	12.0	12.6	6.4	9	7.3	1.2	11.1
20	5	18.3	19.2	11	6	17.8	8.3	18.6
25	4	9.8	8.4	10.9	3	11.1	6.1	12.6
30	1	15	20.5	8.1	0	11.4	14	7.5

Individual (without collaboration) solutions always optimal

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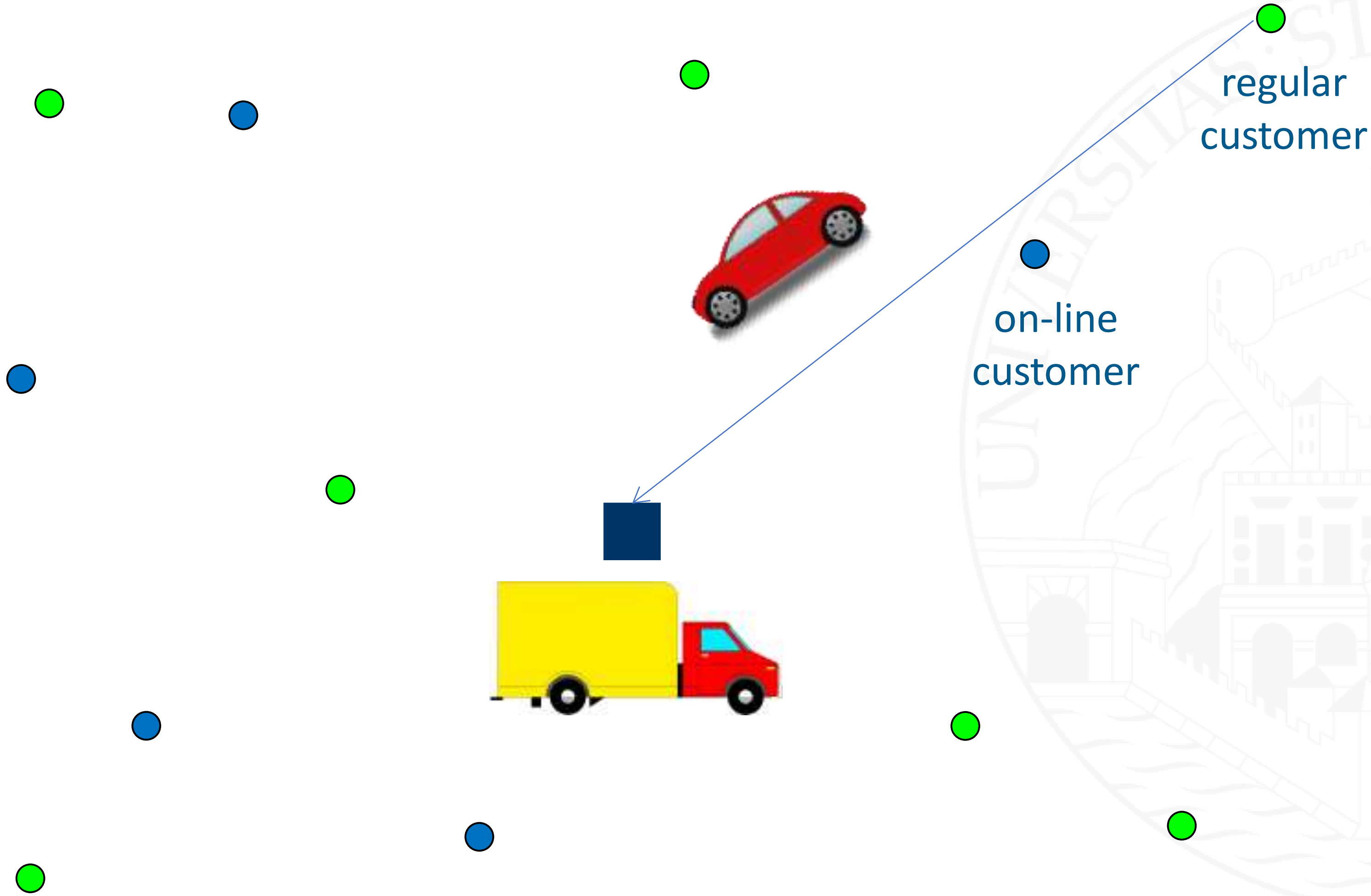
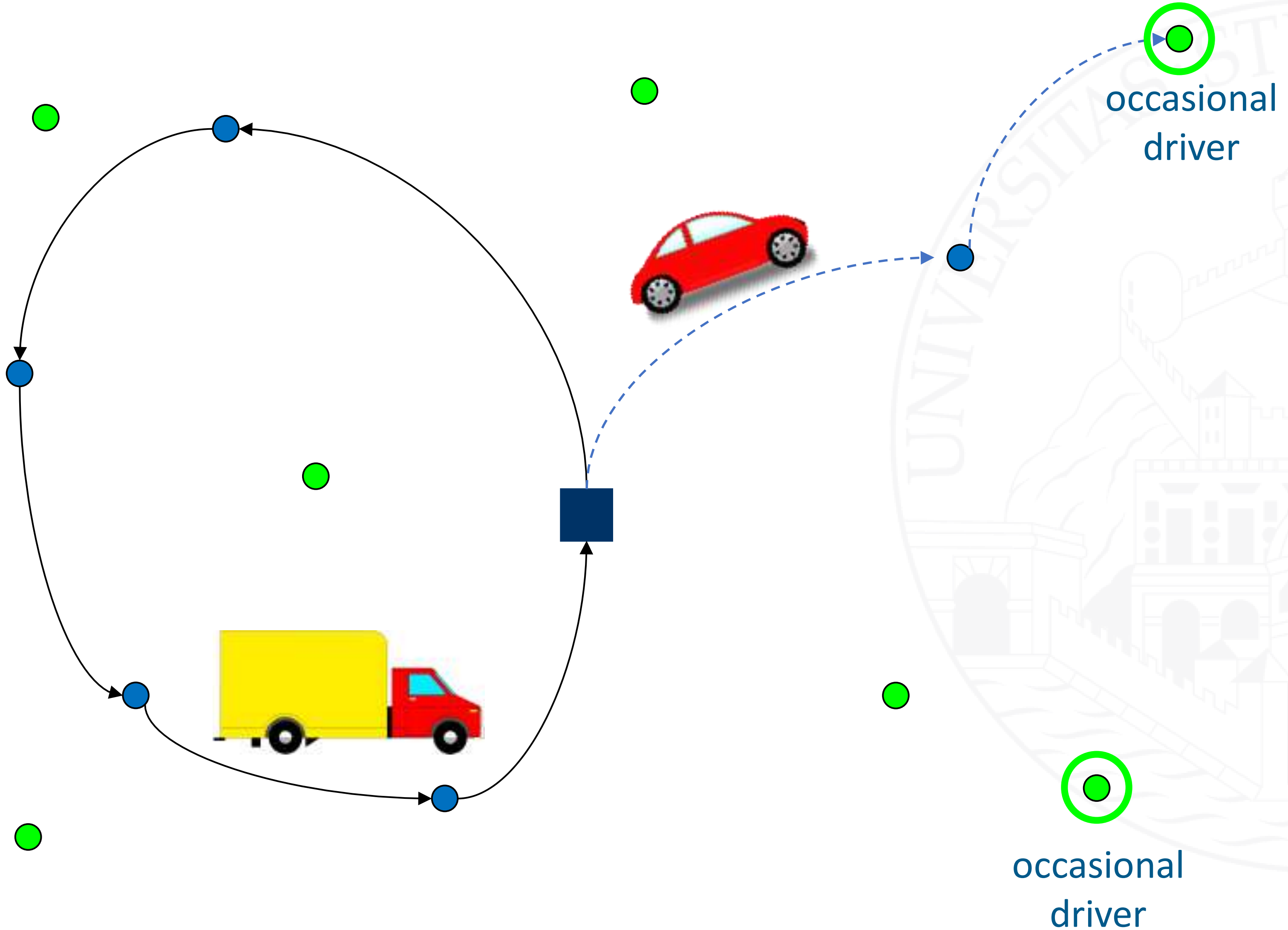
Crowd driving: Occasional drivers



Archetti, Savelsbergh, Speranza, EJOR, 2016

Occasional drivers



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Occasional drivers

Costs:

- Routing cost for regular drivers
- Compensation to occasional drivers

Objective:

Minimize the sum of the cost of regular drivers (routing cost) and occasional drivers (compensation)

Occasional drivers

- Behaviour of occasional drivers
- Compensation schemes
- Objective: Minimize the sum of the cost incurred by regular drivers (routing cost) and occasional drivers (compensation)

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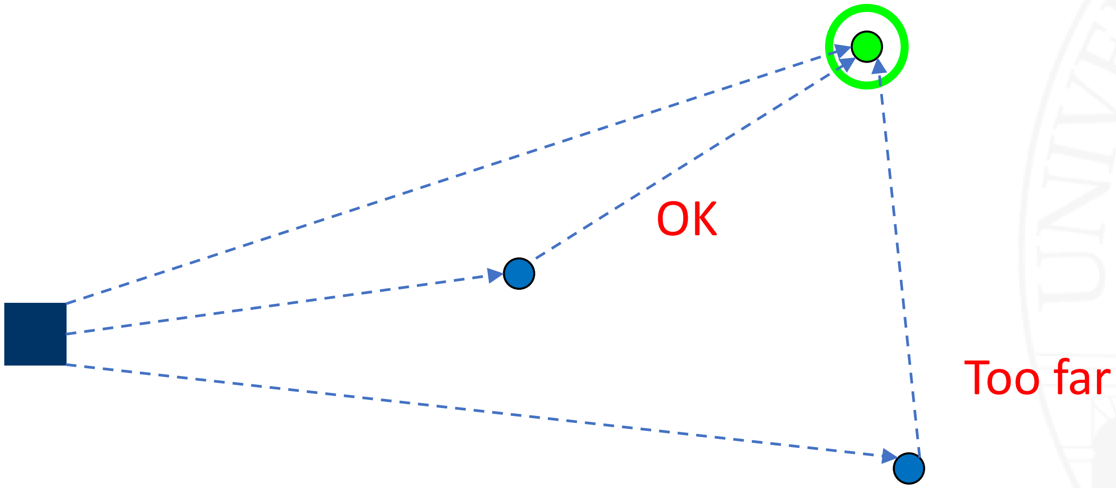
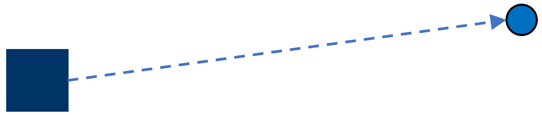
Behaviour of occasional drivers

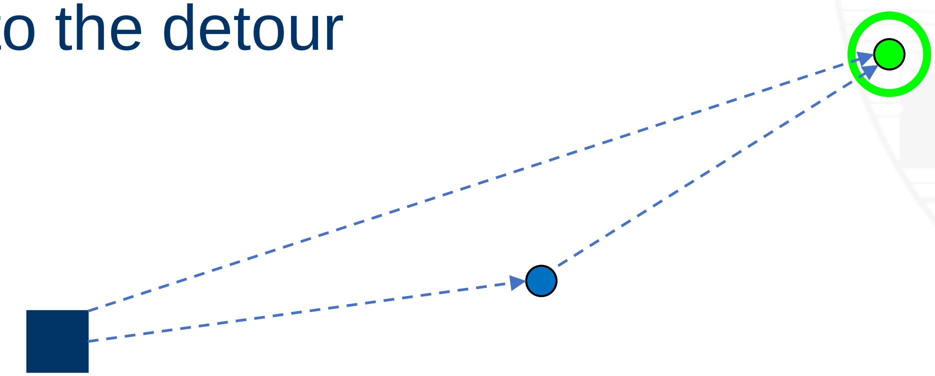


Compensation schemes

1. Proportional to the distance of the on-line customer



2. Proportional to the detour



VRP with occasional drivers

$$\min \sum_{(i,j)\in A} c_{ij} x_{ij} + \sum_{i\in C} \sum_{k\in K} p_{ik} w_{ik}$$ depends on compensation scheme

$$\sum_{j|(i,j)\in A} x_{ij} = \sum_{j|(j,i)\in A} x_{ji} = z_i \qquad i \in C$$

$$\sum_{j|(0,j)\in A} x_{0j} - \sum_{j|(j,0)\in A} x_{j0} = 0$$

$$\sum_{j|(j,i)\in A} y_{ji} - \sum_{j|(i,j)\in A} y_{ij} = \begin{cases} d_i z_i & i \in C \\ \sum_{i\in C} - d_i z_i & i = 0 \end{cases}$$

$$y_{ij} \le Q x_{ij} \qquad (i, j) \in A$$

$$y_{i0} = 0 \qquad i \in C$$

$$w_{ik} \le \beta_{ik} \qquad i \in C, k \in K$$

$$\sum_{i\in C} w_{ik} \le 1 \qquad k \in K$$

$$\sum_{k\in K} w_{ik} + z_i = 1 \qquad i \in C$$

- Exact
- Matheuristic

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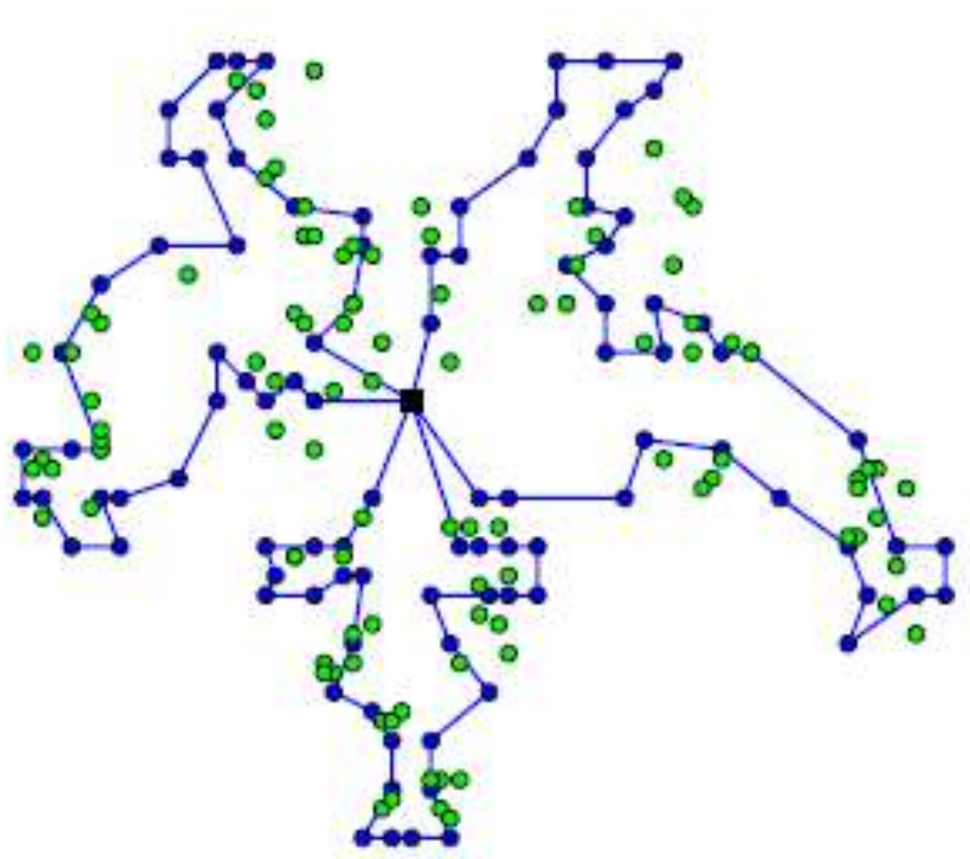
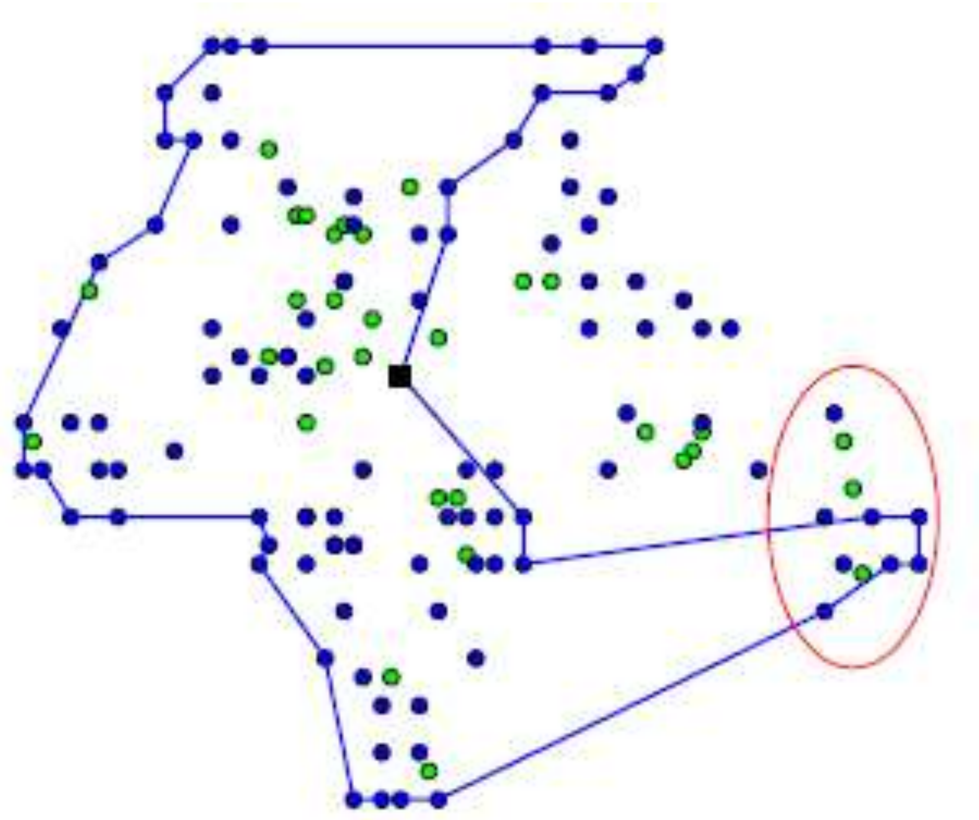
VRP with occasional drivers

Without occasional drivers



VRP with occasional drivers

Compensation scheme proportional to detour



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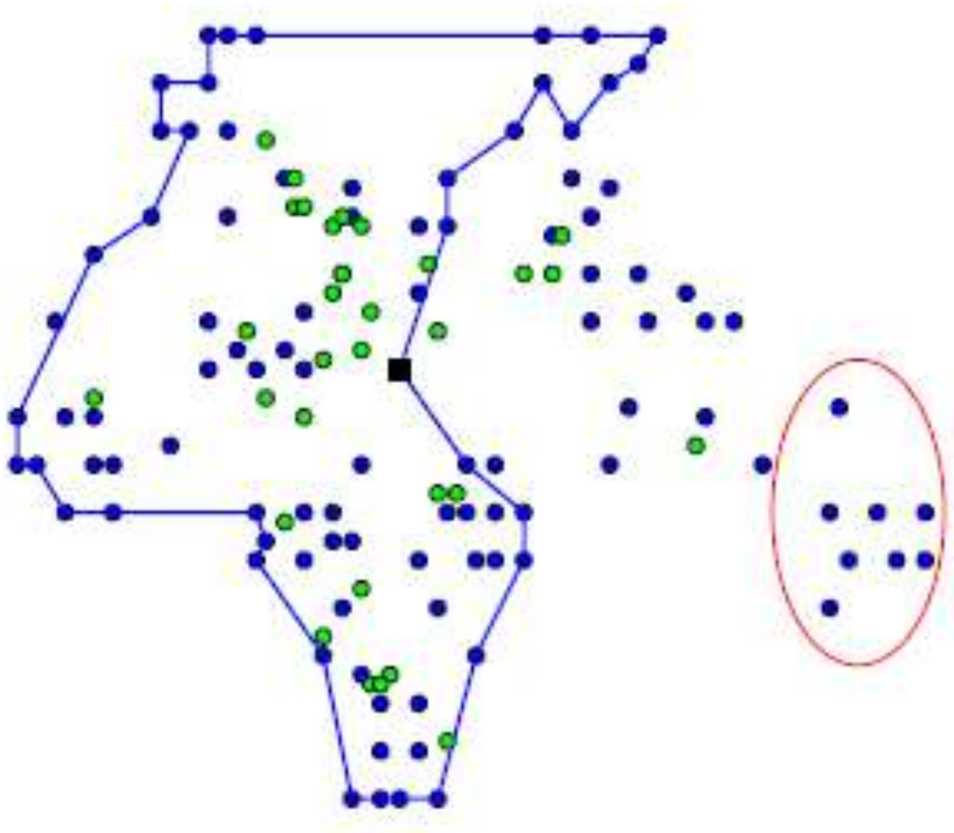
VRP with occasional drivers

Compensation scheme proportional to detour

(lower compensation)



Savings

Compensation scheme proportional to detour

	% cost reduction w.r.t. VRP	% routes reduction w.r.t. VRP	%OD used	% OD cost w.r.t. total cost
C101	43.85	71.67	85.50	35.36
C201	20.49	50.00	66.42	17.04
R101	40.79	64.17	74.92	21.27
R201	33.70	50.00	71.32	20.58
RC101	33.47	52.96	64.20	14.70
RC201	30.05	50.00	61.56	14.28
$\|K\|=50$	26.85	48.07	80.73	12.31
$\|K\|=100$	40.60	64.86	63.77	28.77
$\varsigma=1.1$	31.66	54.58	67.58	14.51
$\varsigma=1.2$	33.16	56.10	69.40	17.11
$\varsigma=1.3$	34.27	56.87	71.60	23.09
$\varsigma=1.4$	34.74	57.56	73.10	23.82
$\varsigma=1.5$	34.80	57.21	72.40	24.16
$\rho=1.2$	34.86	56.70	72.67	20.48
$\rho=1.4$	33.69	57.12	72.72	20.53
$\rho=1.6$	32.63	55.58	63.70	20.60
Average	**33.72**	**56.47**	**70.75**	**20.54**

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Conclusions

Our models and methods

- evolve with the technology
- contribute to the technology

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